• Homework 2.5 $\beta(t) = (t+3, t^2-1, 2)$

(a) Shape. $x = t+3 \Rightarrow t = x-3$, $y = (x-3)^2 - 1$ parabola in $z=2$ plane.

(b) Unit tangent $T = \frac{\beta'(t)}{|\beta'(t)|} = \frac{(1, 2t, 0)}{\sqrt{1+4t^2}} = \left(\frac{1}{\sqrt{1+4t^2}}, \frac{2t}{\sqrt{1+4t^2}}, 0\right)$.

(c) Acceleration $\beta''(t) = (0,2,0)$

Note: Gabe's Thm: This is $(|\beta'(t)|)'$!

(d) Tangential Component of Acceleration $= \beta''(t) \cdot T$

$= (0,2,0) \cdot \left(\frac{1}{\sqrt{1+4t^2}}, \frac{2t}{\sqrt{1+4t^2}}, 0\right) = \frac{4t}{\sqrt{1+4t^2}}$.

(e) For what t is speed increasing? (i.e. tangential component of acceleration is positive). $\frac{4t}{\sqrt{1+4t^2}} > 0$, ie. $\boxed{t>0}$.

(f) Arclength $t=-1$ to $t=2$

$\int_{-1}^{2} |\beta'(t)|\,dt = \ldots \int_{-1}^{2} \sqrt{1+4t^2}\,dt = 6.126$

sagemath: f(t)=sqrt(1+4*t^2) show(integral(f(t),t,-1,2)).n()

Problem: Suppose $\alpha'(t) = \left(\frac{1}{1+t^2}, \cot(t), 2^t\right)$. If $\alpha(0) = (4,1,3)$, find $\alpha(t)$. $\Rightarrow \alpha(t) = \left(\arctan(t) + C_1, \ln|\sin(t)| + C_2, \frac{1}{\ln 2} 2^t + C_3\right)$

$\int \cot(t)\,dt = \int \frac{\cos(t)}{\sin(t)}\,dt = \int \frac{1}{u}\,du = \ln|u| + C_2$
$= \ln|\sin(t)| + C_2$

$u = \sin(t)$
$du = \cos(t)\,dt$

$\Rightarrow \alpha(0) = (4,1,3) = \left(C_1, \text{aaahh..}, \frac{1}{\ln 2} + C_3\right)$

$\Rightarrow C_1 = 4, \ \frac{1}{\ln 2} + C_3 = 3$

Aah $= 1$

$\Rightarrow C_1 = 4, \ C_3 = 3 - \frac{1}{\ln 2}$.

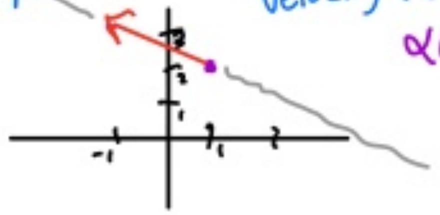
$\boxed{\alpha(t) = \left(\arctan(t) + 4, \ \cancel{\ln|\sin(t)|+} \text{Aauh}, \ \frac{1}{\ln 2} 2^t + 3 - \frac{1}{\ln 2}\right)}$

Sorry — this is impossible.

Thinking.

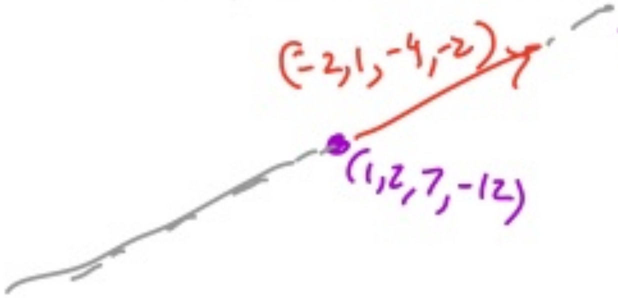
① Graph $\alpha(t) = (1-2t, 2+t)$ in $\mathbb{R}^2$ with velocity vector marked.

$\alpha(t) = (1,2) + t(-2,1)$



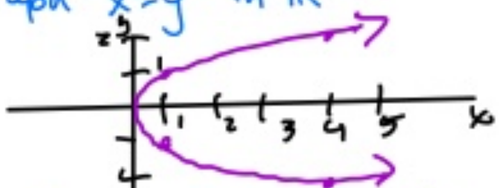
② Graph (schematically) $\gamma(t) = (1-2t, 2+t, 7-4t, -12-2t)$ in $\mathbb{R}^4$

$= (1,2,7,-12) + t(-2,1,-4,-2)$

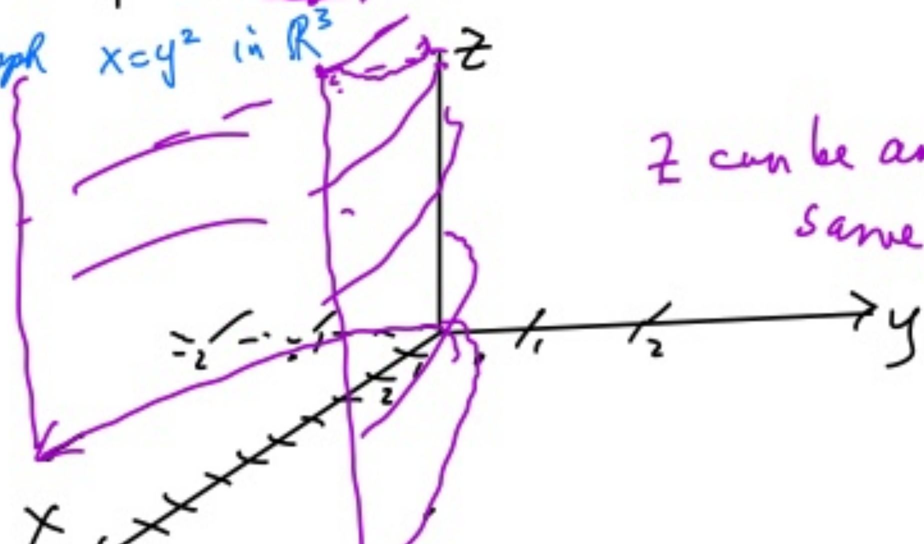


line
$\underbrace{(p_1, p_2, \dots)}_{\text{point on line}} + t \underbrace{(v_1, v_2, \dots)}_{\text{velocity vector}}$

③ Graph $x = y^2$ in $\mathbb{R}^2$.



④ Graph $x = y^2$ in $\mathbb{R}^3$



z can be anything, same xy plot.

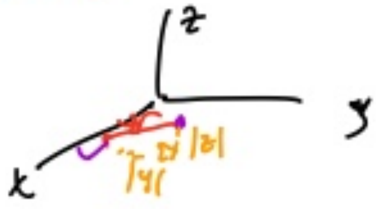
⑤ What does $y^2 + z^2$ mean in the yz plane?

[Distance from (y,z) to $(0,0)$]^2

$r^2 = z^2 + y^2$

⑥ What does $y^2 + z^2$ mean in $\mathbb{R}^3$?

[Distance from (x,y,z) to the x-axis.]^2

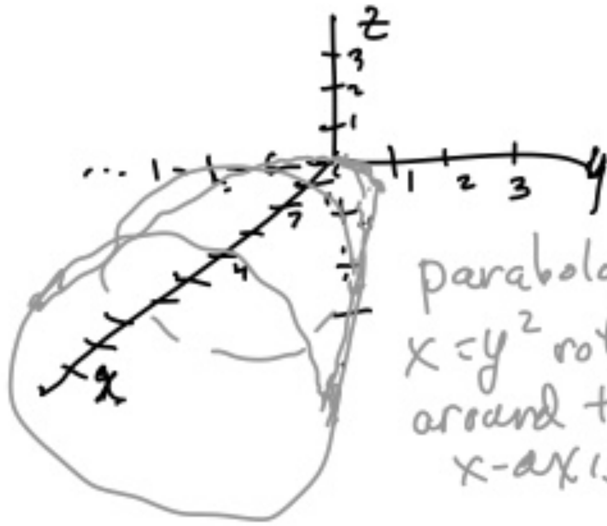


⑦ Graph $x = y^2 + z^2$ in $\mathbb{R}^3$

$0 = y^2 + z^2$

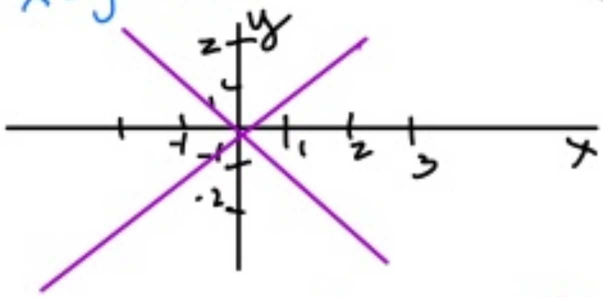
$1 = y^2 + z^2$

$r^2 = y^2 + z^2$



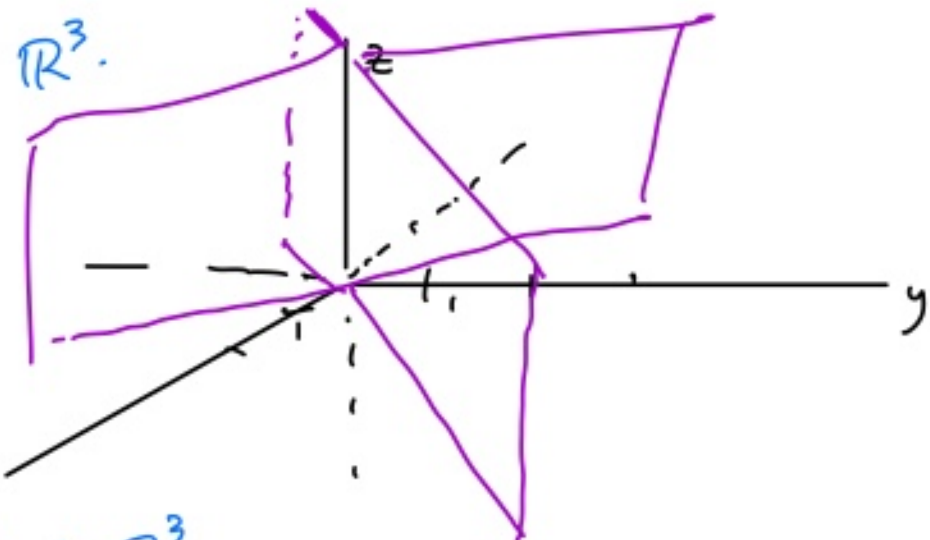
paraboloid.
$x = y^2$ rotated around the x-axis.

⑧ Graph $x^2 = y^2$ in $\mathbb{R}^2$.

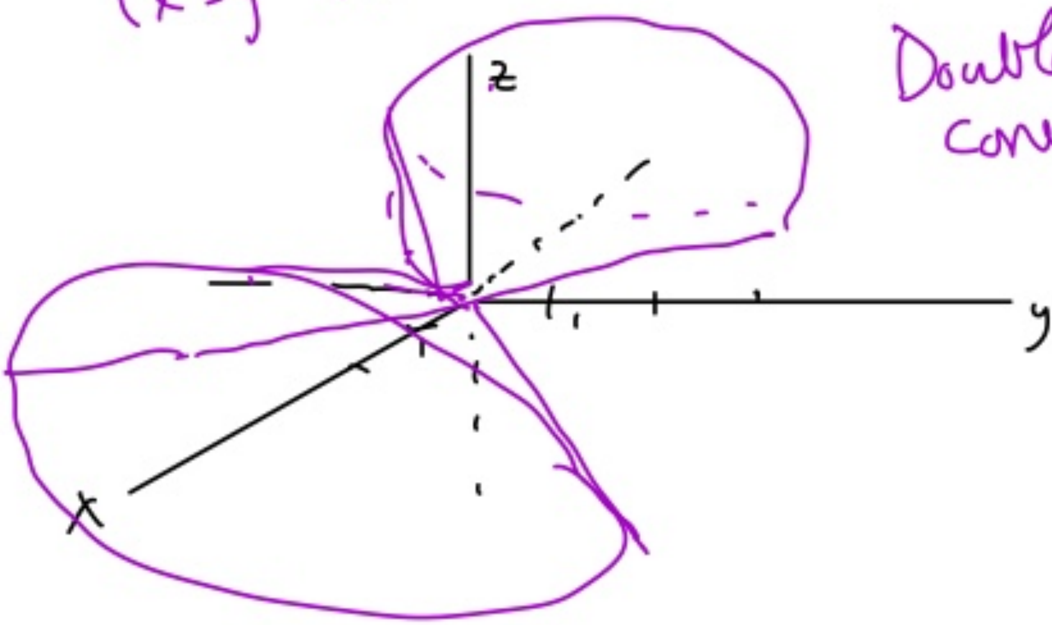


$y = \pm x$

⑨ Graph $x^2 = y^2$ in $\mathbb{R}^3$.



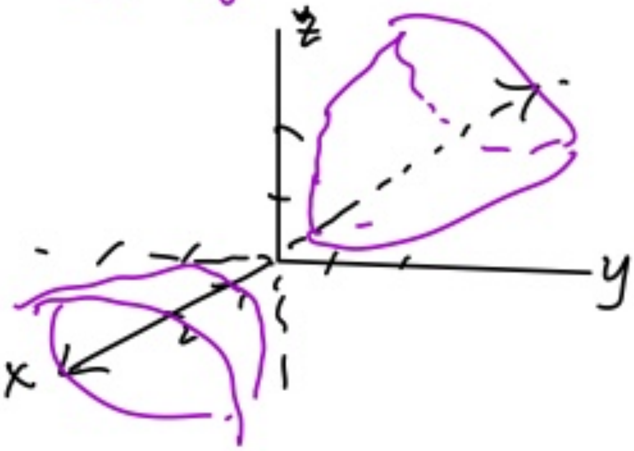
⑩ Graph $x^2 = y^2 + z^2$ in $\mathbb{R}^3$.
($x^2 = y^2$ rotated around the x-axis)



Double cone.

(11) Graph $x^2=y^2+z^2+1$ in $\mathbb{R}^3$.

$\iff y^2+z^2=x^2-1$



$x=0$ $\emptyset$

$x=\pm1$ $\bullet\ (0,0)$

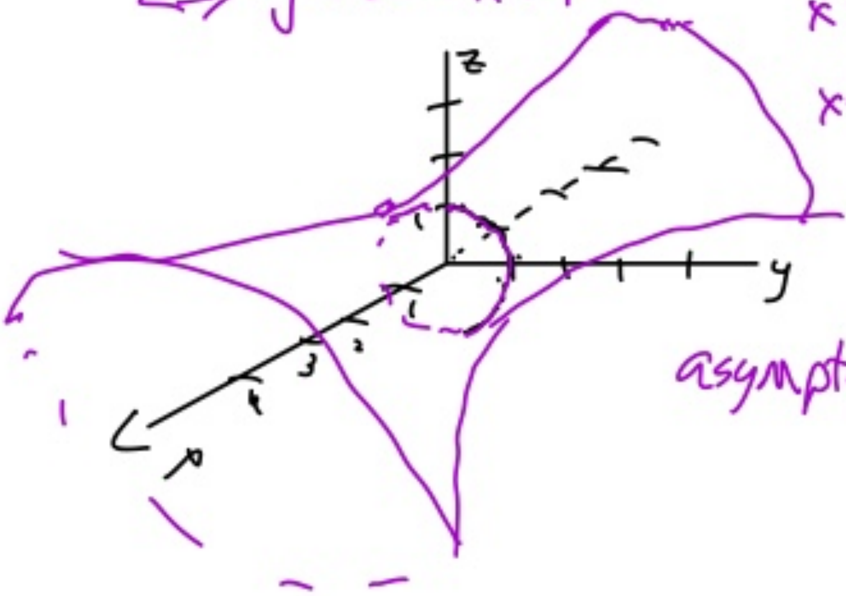
$x=\pm2$ $\sqrt{3}$

$x=\pm3$ $\sqrt{8}$

hyperboloid of 2 sheets, asymptotic to last graph $x^2=y^2+z^2$.

(12) Graph $x^2=y^2+z^2-1$ in $\mathbb{R}^3$.

$\iff y^2+z^2=x^2+1$



$x=0$

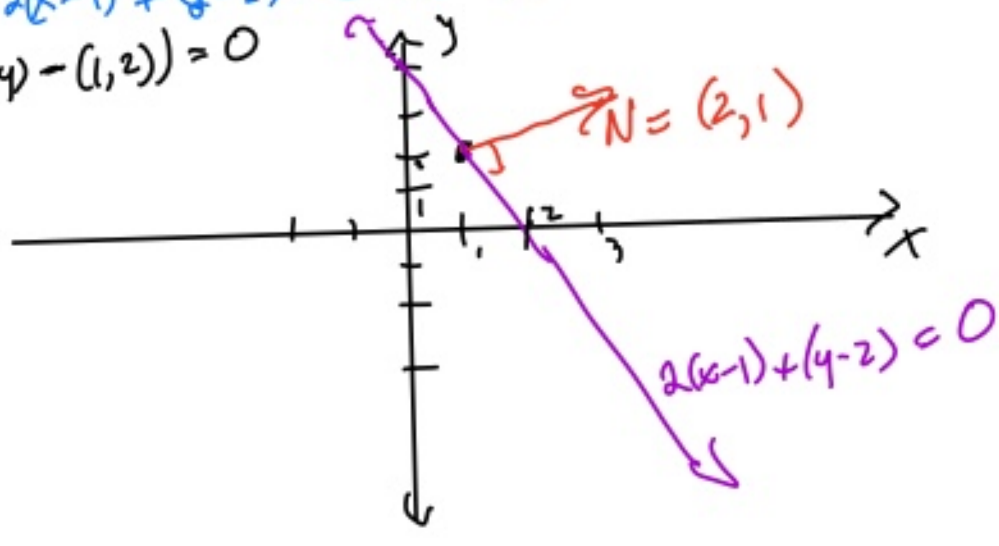
$x=\pm1$ $\sqrt{2}$

$x=\pm2$ $\sqrt{5}$

hyperboloid of 1 sheet, asymptotic to cone in #10, $x^2=y^2+z^2$.

(13) Graph $2(x-1)+(y-2)=0$ in $\mathbb{R}^2$. Mark normal vector

$(2,1)\cdot((x,y)-(1,2))=0$



(19) Graph $2(x-1)+(y-2)+7(z+3)-2(w-1)=0$ in $\mathbb{R}^4$ (schematically)

Normal $(2,1,7,-2)$

point $(1,2,-3,1)$.